PhD position:
Data Assimilation of Systems with Hysteresis. Application to 3D Printing.

Supervisors:

- Prof. Olga Mula (TU Eindhoven, Mathematics & Computer Science, research website),
- Prof. Rob Wolfs (TU Eindhoven, Structural Engineering research website)

Location: TU Eindhoven, Mathematics & Computer Science, CASA, [website]
Duration: 4 years.
How to apply: Please, send your CV to O. Mula along with a short motivation letter, and a list of your MsC courses and grades.
Deadline to apply: December 1st, 2023.
Starting date: January-April 2023.

Project's topic

The goal of the PhD is to develop and analyse data assimilation, and control schemes for systems presenting strong hysteresis. Hysteresis is the dependence of the state on its history. It occurs in ferromagnetic and ferroelectric materials, as well as in the deformation of materials and many other natural phenomena. It is often associated to irreversible microscopic changes in the system that are typically very nonlinear, and difficult to model. One concrete consequence of this is that doing data assimilation, and control of these systems is very challenging. This is because, to reconstruct the state of the system from partial observations, one needs to carefully take into account what happened in the past. To control the system, one needs to take the past into account, and anticipate the hysteresis effects that may occur in the future. Currently, numerous data assimilation schemes tend to be quite agnostic about the past, and there is need for more elaborate schemes.

To develop data assimilation, and control schemes for theses processes, we will assume that the system is described by a time-dependent PDE evolution involving a modeling of hysteresis effects. We will reduce the complexity of solving the forward model with respect to changes in parameters by model order reduction techniques. We will then leverage these techniques to build a data-assimilation scheme to compute reconstructions at a reduced computational cost, and that take into account the memory of the process. To control the system, we will resort to reinforcement learning techniques.

One modern application involving systems with strong hysteresis is 3D printing (aka additive manufacturing). This technology plays nowadays a key role to have a more sustainable production chain. It builds three dimensional objects by depositing material layer after layer. It has enormous potential to build complex shapes with a minimal amount of waste material. It also reduces the amount of dedicated tools needed for production, and the construction can be done locally. One of the main challenges faced by the technology is that the machine instructions to obtain a given object with optimal shape and optimal mechanical properties are still discovered by many trial and error prints. This is, in part, because discovering the machine instructions for a given geometry is a complicated, rather unexplored, control problem of a system with hysteresis where, in addition, geometry evolves over time. Mathematically, the goal is to find an optimal path to deposit the material with the 3d printer. However, the deposition of material induces residual stresses that can result in distortions. These distortions can manifest either immediately or in a delayed manner, and in both cases, they can make the

final product unacceptable because of a bad shape or because of bad mechanical properties. We propose to addess this problem as a test case for the numerical schemes that we will develop.

What will you learn?

A project of this type will allow you to learn a number of topics and skills:

- Data assimilation and control
- Reduced Modeling for State and Parameter Estimation
- Path optimization, topology optimization
- Reinforcement learning (mathematical foundations and implementation)
- You will also learn about the physics, and mechanics of additive manufacturing, and how to interact with colleagues with different backgrounds (mathematics, mechanical engineering)

What are the concepts that you already need to know before starting such a project?

The project is at the interface between computational PDEs, machine learning and mechanical engineering so you must have a taste for interdisciplinary work, and some basic knowledge in numerical methods for PDEs, and machine learning. We will work with mathematical concepts for which there are advantages in describing them in an infinite dimensional setting. In other words, familiarity with the setting of Functional Analysis would definitely be a plus but it is not strictly necessary. Finally, some familiarity with probability theory and optimization methods will be desirable to understand the mathematics of reinforcement learning. To summarize, the requirements are:

- Scientific Computing
- Some knowledge in optimization and machine learning
- Good programming skills (preferably in Python)
- Some knowledge in Applied Functional Analysis, PDEs, and probability is a plus.

Location and supervisors

The PhD will be supervised by Prof. Olga Mula within the Mathematics and Computer Science Department of TU Eindhoven. Prof. Rob Wolfs, from the department of structural engineering, will be a co-supervisor. He will help with modeling aspects of 3D printing, and access to real data.

Terms and conditions

The terms of employment are in accordance with the Dutch collective labor agreement. The groos monthly salary is around 2.720 euros during the first year, and it increases to 3.484 euros over the four year period. Employees are also entitles to a holiday allowance of 8% of the gross annual salary and a year-end bonus of 8.33%.

Bibliography

Some possible references connected to the topics of this PhD are [GLM22, BAT23]. A short introduction to the mathematics of reinforcement learning: here.

References

[BAT23] Mathilde Boissier, Grégoire Allaire, and Christophe Tournier, *Concurrent shape optimization of the part and scanning path for powder bed fusion additive manufacturing*, SIAM Journal on Control and Optimization **61** (2023), no. 2, 697–722.

[GLM22] F. Galarce, D. Lombardi, and O. Mula, *State estimation with model reduction and shape variability. application to biomedical problems*, SIAM Journal on Scientific Computing **44** (2022), no. 3, B805–B833.